BRIEF COMMUNICATIONS

THE PROBLEM OF DEFINING THE CONCEPTS OF RELAXATION TIME AND OF QUASI-STEADINESS OF A PROCESS

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Inzhenerno-Fizicheskii Zhurnal, Vol. 13, No. 4, pp. 560–563, 1967

UDC 536.5

By considering the fluctuations in a system it is shown that it is possible to introduce the concept of time of total cessation of the relaxation process and to determine the value of the limit velocity of a monotonic process below which the process becomes quasi-steady.

In studying any process of establishing the steady state, it is generally necessary to operate with a concept of relaxation time τ, which, as is well known, is the time interval on elapse of which the initial perturbation is reduced by a factor of e. In addition, particularly in textbook literature, it is stressed that the mathematical process of relaxation is concluded after an infinitely great interval of time. At the same time it is obvious that the relaxation ceases within a completely predictable finite time. Moreover, the time of total cessation of relaxation is occasionally employed in calculations [1], although no clear definition of this concept exists. It seems to us that a rigorous physical foundation for this quantity can be given and it should be possible to indicate a method for finding this quantity by giving consideration to the fluctuations in a system. We note that other purely technical factors are possible (for example, the sensitivity of the measuring apparatus employed) which would lead to the apparent cessation of the relaxation process. In the following we will assume that no such factors are present, i.e., we will assume that the recording of the relaxation parameter is accomplished with the necessary accuracy and that the fluctuations serve as the only source of deviations in its value from the magnitudes predicted by macroscopic theory.

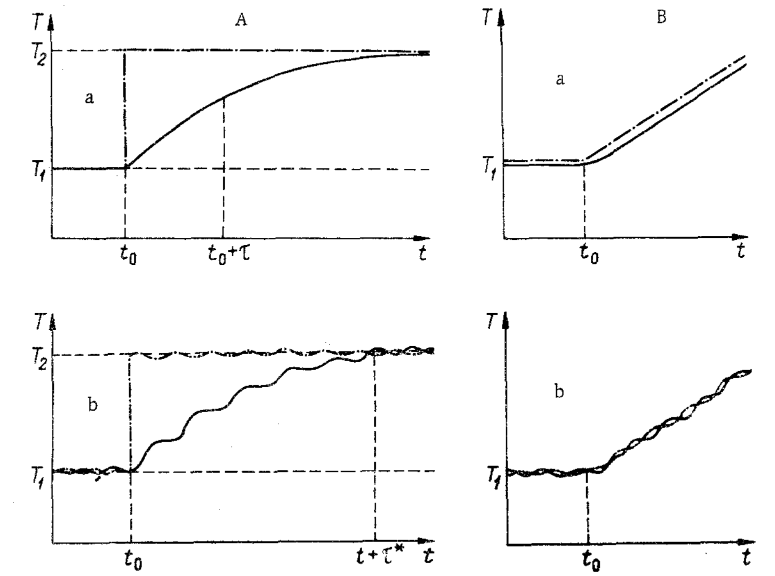
Let us consider this problem on the specific example of the temperature relaxation of a metal solid in a gas medium. Here the change in the temperature of the body is described by a differential equation of the form

$$\frac{dT}{dt} = \frac{\alpha}{C}\,[T_e - T]. \tag{1}$$

With a discontinuous change in the temperature of the ambient gas from T_1 to T_2, the solution of Eq. (1) leads to an exponential change in the temperature of the body with time:

$$T = T_2 - (T_2 - T_1)\ \exp\,[-(t - t_0)/\tau]. \tag{2}$$

The curves for this case are given schematically in the figure (A, a). As a matter of actual fact, no such smooth curves of temperature variation exist. The existence of fluctuations leads to the appearance of unique undulation in each of the curves T = T(t). The real time relationships of the temperatures of the me-



Dependence of external medium temperature (dash-dot line) and body temperature (solid line) on time without account for (a) and with account for (b) for the case of temperature relaxation of a solid in a gas medium and discontinuous change in gas temperature (A) and for the case of a linear change in external medium temperature (B).

dium and the body are shown in somewhat exaggerated scale in the figure (A, b).

The "interlacing" of the curves $T_e = T_e(t)$ and $T = T(t)$ should in actuality obviously correspond to the equality of body and medium temperatures in the ordinary sense. It is easy to see that after the jumpwise change in T_e both of the curves will again begin to "interlace" on passage of a completely defined finite interval of time τ^* which may be referred to as the time of total cessation of relaxation. The quantitative evaluation of τ^* can be carried out if we take into consideration the fact that the fluctuations in T_e are very much smaller than the fluctuations in T, and if we assume that the instant of time $t_0 + \tau^*$ corresponds to equality of the values of $\overline{((\Delta T)^2)}^{1/2} = (kT^2/C)^{1/2}$ [2, 3] to the quantity $T_2 - T$ calculated from formula (2), i.e.,

$$(T_2 - T)\exp[-\tau^*/\tau] = T_2\sqrt{R/NC}, \tag{3}$$

whence

$$\tau^* = \tau[\ln\sqrt{NC/R} + \ln(T_2 - T_1)/T_2] \approx \approx \tau[\ln\sqrt{N} + \ln(T_2 - T_1)/T_2] \approx \approx \tau[55 + \ln(T_2 - T_1)/T_2]. \tag{4}$$

It is immediately clear, first of all, that the time for the total cessation of the process of relaxation is tens of times greater than the usual time of relaxation and, secondly, that the time τ^*, as was to be expected, is a function of the magnitude of the initial deviation from the finite steady state.

It is curious to note that the effect of the initial deviation $(T_2 - T_1)$ under ordinary conditions is insignificant and is virtually always

$$\tau^* \approx 55\tau. \tag{5}$$

Consideration of the fluctuations makes it possible also to formulate the quantitative criterion defining the quasi steadiness of the process. We will deal with this process also in the specific example of a monotonic (precisely linear) change in the temperature of the external medium in the model indicated above.

Let the temperature of the ambient gas begin to increase from a certain instant t_0 according to the law

$$T_e = T_1 + \gamma t. \tag{6}$$

Differential equation (1) then assumes the form

$$\frac{dT}{dt} = \frac{1}{\tau}[T_1 + \gamma t - T] \tag{7}$$

and its solution will be

$$T = T_e - \gamma\tau\{1 - \exp[-(t - t_0)/\tau]\}. \tag{8}$$

The curves of the functions $T_e = T_e(t)$ and $T = T(t)$ are shown schematically in the figure (B, a). For long periods of time $[(t - t_0) \gg \tau]$ we can write

$$T = T_e - \gamma\tau,$$

i.e., there is a certain lag in the temperature of the body from the temperature of the medium. Apparently, as in the case considered above, we may assume that the temperature of the body and that of the medium are identical if

$$T_e - T = \gamma\tau \leqslant \sqrt{\overline{(\Delta T)^2}}, \tag{9}$$

i.e., the curves of the two functions "interlace" all of the time. Consequently, the condition for the quasi steadiness of the process will be the following relationship:

$$\gamma\tau \leqslant T\sqrt{R/NC}. \tag{10}$$

Thus the rate of change in temperature should not exceed the quantity

$$\gamma = \frac{dT}{dt} = \frac{T}{\tau}\sqrt{\frac{R}{NC}} \approx \frac{T}{\tau}\frac{1}{\sqrt{N}} \approx \frac{T}{\tau}\frac{1}{8\cdot 10^{11}}. \tag{11}$$

If the periodic change in the external-medium temperature ($T_e = T_0 + T_{AB}\sin\omega t$), quasi steadiness will be achieved at the frequencies

$$\omega\tau < \frac{1}{\sqrt{N}} \approx 10^{-12}. \tag{12}$$

The calculations of the time of total cessation of relaxation and of the maximum rate of the quasi-steady process, carried out on the specific model, are primarily of theoretical interest both from a fundamental standpoint and from the point of view of processing the measurement results (for example, the calorimetric results). The quantitative determinations of the quasi steadiness of the process and of the time of total relaxation must be taken into consideration in designing technological processes. Calculations of this type may be carried out for any specific cases.

NOTATION

T is the body temperature; T_e is the external medium temperature; t is time; α is the heat transfer coefficient; C is the heat capacity of a body; τ is the relaxation time; τ^* is the time of complete cessation of relaxation; N is the Avogadro number; R is the universal gas constant; γ is the rate of a monotonic temperature change; T_0 is the mean temperature of a medium in case of its periodic change; T_{AB} is the maximum deviation of external medium temperature from its mean value (amplitude); ω is the cyclic frequency.

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